term dominates, the phugoid frequency works out to be approximately independent of speed. It is possible to draw similar conclusions with the more complicated expression for phugoid frequency used in Ref. 1.

We now seek data from the literature in support of the conclusions in case 2. In practice, the nondimensional aerodynamic and thrust derivatives vary with speed, especially in the transonic regime. We, therefore, look for data at low Mach numbers, where the aerodynamic and thrust derivatives may be expected to be approximately constant. A perusal of the data in Ref. 3 provides several examples, one each of which illustrates the two conclusions arrived at earlier for case 2 and are cited in Table 1. In case of airplane a, the phugoid frequency shows a decrease with increase in speed, whereas for airplane b, the phugoid frequency is nearly constant with increase in speed.

IV. Conclusions

It is shown that the variation of the phugoid frequency with speed depends on the manner in which the speed is varied between two level-flight equilibrium states. If C_L is kept fixed as the speed is varied, the phugoid frequency shows the well-known inverse dependence on speed. It is, therefore, incorrect to suggest that the inverse variation of the phugoid frequency with speed is only due to changes in the aerodynamic and thrust derivatives. However, the phugoid frequency does turn out to be independent of speed in certain cases, when the speed is varied with the altitude kept constant.

References

[1]Pradeep, S., and Kamesh, S., "Does the Phugoid Frequency Depend on Speed?" *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 2, 1999, pp. 372, 373.

[2]Roskam, J., *Airplane Flight Dynamics and Automatic Flight Controls*, Vol. 1, Roskam Aviation and Engineering Corp., Lawrence, KS, 1979, p. 6:34.

[3]Schmidt, L. V., *Introduction to Aircraft Flight Dynamics*, AIAA, Reston, VA, 1998, pp. 333–354.

Reply by the Author to N. Ananthkrishnan

S. Pradeep*

Indian Institute of Science, Bangalore 560 012, India

Introduction

IN a recent paper[1], contrary to popular belief, it was shown that the phugoid frequency is independent of the forward speed. This was done by using a new approximation to the phugoid frequency that is very accurate. The research suggested that it was not speed, but the conspired variation of the aerodynamic derivatives with speed that caused the decrease of phugoid frequency with an increase in speed. To support the argument, counterexamples in which the phugoid frequency increased with an increase in speed were shown. This broke a century-old belief and has, quite naturally, stirred up a debate. In a recent note,[2] it is claimed that the results quoted are only partially correct. The misunderstanding has cropped up because the approximation to the phugoid frequency that is used to arrive at the conclusion in the first study is inadequate and requires amelioration. The purpose of this note is to clarify the issues raised.

Clarification

The controversial result[1] is that if the speed alone is varied, keeping the aerodynamic and the thrust derivatives fixed by some means, the phugoid frequency remains unchanged, establishing that the phugoid frequency is independent of speed.

Received 15 May 2000; accepted for publication 14 June 2000.

*Associate Professor, Department of Aerospace Engineering. Member AIAA.

Table 1 Percentage error in the 2DOF phugoid frequency approximation from the exact value

Aircraft	Flight phase	% error
A		−15.02
B	1	−14.38
	2	−17.93
	3	−7.41
C	1	−24.27
	2	−27.52
	3	−19.78
D	1	−12.63
	2	10.50
	3	19.66
E	1	−17.92
	3	33.10
F	1	−17.41
	2	−34.53
	3	−6.53

Ananthkrishnan[2] rephrases the problem in the following manner: "If the equilibrium state of the airplane is changed from one steady, level flight (p_1, U_1, C_{1a}) to another (p_2, U_2, C_{2a}) what is the corresponding change in phugoid frequency?" The two equilibrium states are shown to be related by the following constraint.

$$\tfrac{1}{2}p_1U_1^2SC_{1a} = \tfrac{1}{2}p_2U_2^2SC_{ta} \tag{1}$$

As the speed is varied from U_1 to U_2, both the density and the coefficient of lift change subject to the constraint in Eq. (1). On the basis of this, two special cases, one with the coefficient of lift kept unchanged and the other with the altitude kept unchanged, are examined. Until this point, the argument is flawless. The 2DOF approximation[3] is then used to affirm that the phugoid frequency is inversely proportional to speed.[2] For a study of this nature, the 2DOF approximation, $w_p = gZ_u/U_1$ is inappropraite because it frequently results in large errors. This is the source of the incorrect inferences of Ref. 2.

To validate this claim, Table 1 presents the percentage error in the 2DOF phugoid frequency approximation from the exact value, calculated for the same set of data that was used in former studies by the author.[1] It is taken from Appendix C of Roskam's text on flight dynamics.[4] This collection of data pertains to six modern aircraft in a total of 16 flight conditions. The chosen aircraft possess dissimilar missions: a small four-place transportation airplane, a 19-passenger commuter airliner, a small jet trainer, a medium-sized high performance business jet, a supersonic fighter-bomber, and a large wide-body jet transport. The flight conditions extend from power approach at sea level to cruise at medium and high altitudes. The database is thus representative of a whole gamut of airplanes and flight conditions. In the 15 cases examined, the percentage error between the exact value of the phugoid frequency and that calculated from the approximation varies from 6.53% to 33.10%. It is not surprising that judgments based on this expression are erroneous.

On the contrary, the approximation[3] to the phugoid frequency

$$w_p = \sqrt{\frac{g(M_\alpha Z_u - M_\alpha Z_a)}{M_g Z_a - U_1 M_\alpha}} \tag{2}$$

does not differ from the exact value by more than 4% in the 15 cases considered. Conclusions based on Eq. (2) have, therefore, greater authenticity. Reference 2 also states that similar conclusions can be reached from the nondimensional version of Eq. (2). The nondimensional version derived in Ref. 2 [Eq. (4) of Ref. 2] is incorrect, because U_1 has not been canceled with q_1. The correct expression is[1]

$$w_p = \sqrt{\frac{g\left\{C_{m_0}\left(C_{L_0} + 2C_{L_1}\right) - \left(C_{w_0} + 2C_m\right)\left(C_{l_0} + C_{D_1}\right)\right\}}{\left(C_{m_q}\bar{c}/2\right)\left(C_{L_0} + C_{D_1}\right) + (2m/pS)C_{m_\alpha}}} \tag{3}$$

Table 2 The ratio of C_{ta} to C_{la} when compared with 2

C_{La}	C_{L1}	$(C_{La}/C_L)/2$
0.00	0.31	0.00
0.03	1.15	0.01
0.02	0.19	0.05
0.02	0.30	0.03
0.07	1.40	0.03
0.08	0.15	0.28
0.13	0.23	0.28
0.04	1.64	0.01
0.40	0.41	0.49
0.28	0.28	0.50
0	1.00	0.00
−0.18	0.17	−0.52
−0.22	1.76	−0.06
−0.23	0.52	−0.22
0.13	0.40	0.16

It is clearly seen that the speed, U_1, does not appear explicitly in Eq. (3), proving that the phugoid frequency is independent of the forward speed, provided the aerodynamic and the thrust derivatives are held fixed by some means. Thus, it is the incorrect phugoid expression that has led to the disbelief in the claim that the phugoid frequency is independent of speed.

Apart from the above reasoning, two minor errors in Ref. 2 are pointed out here. First, the arguments in Ref. 2 cannot support the fact that the phugoid frequency could sometimes be proportional to the forward speed, as shown in Table 1 of Ref. 1. This occurs because the aerodynamic derivatives change in such a manner as to increase the phugoid frequency with increase in speed. Second, when C_1 is constant, the following expression is obtained for frequency[2]

$$w_p = (g/U_1)\sqrt{(C_{La}/C_{La}) + 2}$$

The argument that the ratio of the aerodynamic derivatives inside the square root are small in comparison to 2, is not always true, as shown in Table 2.

Conclusion

The widespread belief of the dependency of the phugoid frequency on speed is perhaps because most often the aerodynamic derivatives vary with speed in such a manner as to make the phugoid time period roughly proportional to the speed. If the aerodynamic derivatives are assumed to be held constant by some means, then the phugoid frequency remains invariant with change in speed.

References

[1] Pradeep, S., and Kamesh, S., “Does the Phugoid Frequency Depend on Speed?” *Journal of Guidance, Control, and Dynamics*, Vol. 22. No. 2, 1999, p. 373.

[2] Ananthkrishnan, N., “Comment on Does the Phugoid Frequency Depend on Speed?” *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 4, 2000, pp. 765–766.

[3] Pradeep, S., “A Century of Phugoid Approximations,” *Aircraft Design*, Vol. 1, Pergamon Press, New York, 1998, pp. 89–104.

[4] Rosham, J., *Airplane Flight Dynamics and Automatic Flight Controls*, DAR-Corporation, Lawrence, KS, Pt. 1, 1998.